J. Indones. Math. Soc. (MIHMI)
Vol. 15, No. 2 (2009), pp. 79–89.

NEW SUBCLASS OF UNIVALENT FUNCTIONS DEFINED BY USING GENERALIZED SALAGEAN OPERATOR

S. B. Joshi and N. D. Sangle

Abstract. In this paper, we have introduced and studied a new subclass $TD_\lambda(\alpha, \beta, \xi; n)$ of univalent functions defined by using generalized Salagean operator in the unit disk $U = \{z : |z| < 1\}$ We have obtained among others results like, coefficient inequalities, distortion theorem, extreme points, neighbourhood and Hadamard product properties.

1. INTRODUCTION

Let A denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1}$$

which are analytic in the unit disk $U = \{z : |z| < 1\}$. In [4], Al-oboudi defined a differential operator as follows, for a function $f \in A$,

$$D^0 f(z) = f(z),$$
$$Df(z) = D^1 f(z) = (1-\lambda) f(z) + \lambda z f'(z) = D_\lambda f(z), \lambda \geq 0, \tag{2}$$

in general

$$D^n f(z) = D_\lambda(D^{n-1} f(z)). \tag{3}$$

If $f(z)$ is given by (1), then from (2) and (3) we observe that

$$D^n f(z) = z + \sum_{k=2}^{\infty} [1 + (k-1)\lambda]^n a_k z^k \tag{4}$$

Received 20-06-2008, Accepted 18-09-2009.
2000 Mathematics Subject Classification: 30C45
Key words and Phrases: Univalent function, Distortion theorem, Neighbourhood, Hadamard product

when $\lambda = 1$, get Salagean differential operator [7]. Further, let T denote the subclass of A which consists of functions of the form

$$f(z) = z - \sum_{k=2}^{\infty} a_k z^k, a_k \geq 0. \tag{5}$$

A function $f(z)$ belonging to A is in the class $D_\lambda(\alpha, \beta, \xi; n)$, if and only if

$$\left| \frac{(D^n f(z))' - 1}{2\xi[(D^n f(z))' - \alpha] - [(D^n f(z))' - 1]} \right| < \beta \tag{6}$$

where $0 \leq \alpha < \frac{1}{2}\xi$, $0 < \beta \leq 1$, $1/2 \leq \xi \leq 1$, $n \in N \cup \{0\}$, $z \in U$. Let

$$TD_\lambda(\alpha, \beta, \xi; n) = T \cap D_\lambda(\alpha, \beta, \xi; n). \tag{7}$$

2. MAIN RESULTS

Theorem 1. *Let f be defined by (5). Then $f \in TD_\lambda(\alpha, \beta, \xi; n)$ if and only if*

$$\sum_{k=2}^{\infty} [1 + (k-1)\lambda]^n k[1 + \beta(2\xi - 1)]a_k \leq 2\beta\xi(1 - \alpha) \tag{8}$$

$0 \leq \alpha < \frac{1}{2}\xi$, $0 < \beta \leq 1$, $1/2 \leq \xi \leq 1$, $n \in N \cup \{0\}$, $\lambda \geq 0$.

Proof. For $|z| = 1$, we get

$$\begin{aligned}
&|(D^n f(z))' - 1| - \beta|2\xi[(D^n f(z))' - \alpha] - [(D^n f(z))' - 1]| \\
&\quad = \left| - \sum_{k=2}^{\infty} [1 + (k-1)\lambda]^n k a_k z^{k-1} \right| \\
&\qquad - \beta \left| 2\xi(1 - \alpha) - 2\xi \sum_{k=2}^{\infty} [1 + (k-1)\lambda]^n k a_k z^{k-1} + \sum_{k=2}^{\infty} [1 + (k-1)\lambda]^n k a_k z^{k-1} \right| \\
&\quad \leq \sum_{k=2}^{\infty} [1 + (k-1)\lambda]^n k[1 + \beta(2\xi - 1)]a_k - 2\beta\xi(1 - \alpha) \\
&\quad \leq 0,
\end{aligned}$$

by hypothesis. Thus by maximum modulus theorem, we have $f \in TD_\lambda(\alpha, \beta, \xi; n)$.

Conversely, suppose that $f \in TD_\lambda(\alpha, \beta, \xi; n)$ hence the condition (6) gives us

$$\begin{aligned}
&\left| \frac{(D^n f(z))' - 1}{2\xi[(D^n f(z))' - \alpha] - [(D^n f(z))' - 1]} \right| \\
&\qquad = \left| \frac{- \sum_{k=2}^{\infty} [1 + (k-1)\lambda]^n k a_k z^{k-1}}{2\xi(1 - \alpha) - (2\xi - 1) \sum_{k=2}^{\infty} [1 + (k-1)\lambda]^n k a_k z^{k-1}} \right| < \beta.
\end{aligned}$$

Since $|Re(z)| < |z|$ for all z, we obtain

$$Re\left\{\frac{-\sum_{k=2}^{\infty}[1+(k-1)\lambda]^n k a_k z^{k-1}}{2\xi(1-\alpha)-(2\xi-1)\sum_{k=2}^{\infty}[1+(k-1)\lambda]^n k a_k z^{k-1}}\right\} < \beta.$$

Letting $z \to 1^-$ through real values, we get (8). The result is sharp for the function

$$f(z) = z - \frac{2\beta\xi(1-\alpha)}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]} z^k, k \geq 2.$$

Corollary 1. *Let $f \in T$ belong to the class $TD_\lambda(\alpha, \beta, \xi; n)$ then*

$$a_k \leq \frac{2\beta\xi(1-\alpha)}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}, k \geq 2. \tag{9}$$

Theorem 2. *Let $f \in T$ belong to the class $TD_\lambda(\alpha, \beta, \xi; n)$, then for $|z| \leq r < 1$, we have*

$$r - r^2\frac{\beta\xi(1-\alpha)}{1+\beta(2\xi-1)} \leq |D^n f(z)| \leq r + r^2\frac{\beta\xi(1-\alpha)}{1+\beta(2\xi-1)} \tag{10}$$

and

$$1 - r\frac{2\beta\xi(1-\alpha)}{1+\beta(2\xi-1)} \leq |(D^n f(z))'| \leq 1 + r\frac{2\beta\xi(1-\alpha)}{1+\beta(2\xi-1)}. \tag{11}$$

bounds given by (10) and (11) are sharp.

Proof. By Theorem 1, we have

$$\sum_{k=2}^{\infty}[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]a_k \leq 2\beta\xi(1-\alpha)$$

then, we have

$$2(1+\lambda)^n[1+\beta(2\xi-1)]a_k \leq \sum_{k=2}^{\infty}[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]a_k \leq 2\beta\xi(1-\alpha),$$

then,

$$\sum_{k=2}^{\infty} a_k \leq \frac{2\beta\xi(1-\alpha)}{2(1+\lambda)^n[1+\beta(2\xi-1)]}$$

Hence

$$\begin{aligned}
|D^n f(z)| &\le |z| + \sum_{k=2}^{\infty} \left|[1+(k-1)\lambda]^n a_k z^k\right| \\
&\le |z| + |z|^2 (1+\lambda)^n \sum_{k=2}^{\infty} a_k \\
&\le r + r^2 (1+\lambda)^n \sum_{k=2}^{\infty} a_k \\
&\le r + r^2 \frac{\beta\xi(1-\alpha)}{1+\beta(2\xi-1)},
\end{aligned}$$

and

$$\begin{aligned}
|D^n f(z)| &\ge |z| - \sum_{k=2}^{\infty} \left|[1+(k-1)\lambda]^n a_k z^k\right| \\
&\ge |z| - |z|^2 (1+\lambda)^n \sum_{k=2}^{\infty} a_k \\
&\ge r - r^2 (1+\lambda)^n \sum_{k=2}^{\infty} a_k \\
&\ge r - r^2 \frac{\beta\xi(1-\alpha)}{1+\beta(2\xi-1)},
\end{aligned}$$

thus (10) is true. Further,

$$\begin{aligned}
|(D^n f(z))'| &\le 1 + 2r(1+\lambda)^n \sum_{k=2}^{\infty} a_k \\
&\le 1 + r \frac{2\beta\xi(1-\alpha)}{1+\beta(2\xi-1)},
\end{aligned}$$

and

$$\begin{aligned}
|(D^n f(z))'| &\ge 1 - 2r(1+\lambda)^n \sum_{k=2}^{\infty} a_k \\
&\ge 1 - r \frac{2\beta\xi(1-\alpha)}{1+\beta(2\xi-1)}.
\end{aligned}$$

The result is sharp for the function $f(z)$ defined by

$$f(z) = z - \frac{2\beta\xi(1-\alpha)}{1+\beta(2\xi-1)} z^2, z = \pm r.$$

Theorem 3. *Let* $n \in N \cup \{0\}$, $\lambda \geq 0$, $0 \leq \alpha_1 \leq \alpha_2 < \frac{1}{2}\xi$, $0 < \beta \leq 1$, $1/2 \leq \xi \leq 1$. *Then* $TD_\lambda(\alpha_2, \beta, \xi; n) \subset TD_\lambda(\alpha_1, \beta, \xi; n)$.

Proof. By assumption we have

$$\frac{2\beta\xi(1-\alpha_2)}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]} \leq \frac{2\beta\xi(1-\alpha_1)}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}$$

Thus, $f(z) \in TD_\lambda(\alpha_2, \beta, \xi; n)$ implies that

$$\sum_{k=2}^{\infty}[1+(k-1)\lambda]^n a_k \leq \frac{2\beta\xi(1-\alpha_2)}{k[1+\beta(2\xi-1)]} \leq \frac{2\beta\xi(1-\alpha_1)}{k[1+\beta(2\xi-1)]}$$

then $f(z) \in TD_\lambda(\alpha_1, \beta, \xi; n)$.

Theorem 4. *The set* $TD_\lambda(\alpha, \beta, \xi; n)$ *is the convex set.*

Proof. Let $f_i(z) = z - \sum_{k=2}^{\infty} a_{k,i} z^k$ $(i = 1, 2)$ belong to $TD_\lambda(\alpha, \beta, \xi; n)$ and let $g(z) = \zeta_1 f_1(z) + \zeta_2 f_1(z)$ with ζ_1 and ζ_2 non negative and $\zeta_1 + \zeta_2 = 1$, we can write

$$g(z) = z - \sum_{k=2}^{\infty}(\zeta_1 a_{k,1} + \zeta_2 a_{k,2}) z^k.$$

It is sufficient to show that $g(z) = TD_\lambda(\alpha, \beta, \xi; n)$ that means

$$\begin{aligned}
&\sum_{k=2}^{\infty}[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)](\zeta_1 a_{k,1} + \zeta_2 a_{k,2}) \\
&= \zeta_1 \sum_{k=2}^{\infty}[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)] a_{k,1} + \zeta_2 \sum_{k=2}^{\infty}[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)] a_{k,2} \\
&\leq \zeta_1(2\beta\xi(1-\alpha)) + \zeta_2(2\beta\xi(1-\alpha)) \\
&= (\zeta_1 + \zeta_2)(2\beta\xi(1-\alpha)) \\
&= 2\beta\xi(1-\alpha)
\end{aligned}$$

Thus $g(z) \in TD_\lambda(\alpha, \beta, \xi; n)$.

We shall now present a result on extreme points in the following theorem

Theorem 5. *Let* $f_1(z) = z$ *and*

$$f(z) = z - \frac{2\beta\xi(1-\alpha)}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]} z^k$$

for all $k \geq 2$, $n \in N \cup \{0\}$, $\lambda \geq 0$, $0 \leq \alpha < \frac{1}{2}\xi$, $0 < \beta \leq 1$, $1/2 \leq \xi \leq 1$. Then $f(z)$ is in the subclass $TD_\lambda(\alpha, \beta, \xi; n)$ if and only if it can be expressed in the form $f(z) = \sum_{k=2}^{\infty} \gamma_k z^k$ where $\gamma_k \geq 0$ and $\sum_{k=2}^{\infty} \gamma_k = 1$ or $1 = \gamma_1 + \sum_{k=2}^{\infty} \gamma_k$.

Proof. Let $f(z) = \sum_{k=2}^{\infty} \gamma_k z^k$ where $\gamma_k \geq 0$ and $\sum_{k=2}^{\infty} \gamma_k = 1$. Thus

$$f(z) = z - \sum_{k=2}^{\infty} \frac{2\beta\xi(1-\alpha)}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]} \gamma_k z^k$$

and we obtain

$$\sum_{k=2}^{\infty} \left(\frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha)} \gamma_k \times \frac{2\beta\xi(1-\alpha)}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]} \right)$$
$$= \sum_{k=2}^{\infty} \gamma_k = 1 - \gamma_1 \leq 1.$$

In view of Theorem 1, this show that $f(z) \in TD_\lambda(\alpha, \beta, \xi; n)$.

Conversely, suppose that of the form (5) belong to $TD_\lambda(\alpha, \beta, \xi; n)$ then

$$a_k \leq \frac{2\beta\xi(1-\alpha)}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}, k \geq 2,$$

Putting

$$\gamma_k = \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha)}$$

and $\gamma_1 = 1 - \sum_{k=2}^{\infty} \gamma_k$, then we have $f(z) = \gamma_1 f_1(z) + \sum_{k=2}^{\infty} \gamma_k f_k(z)$. This completes the proof.

3. NEIGHBOURHOOD AND HADAMARD PRODUCT PROPERTIES

Definition 1. *[6] Let $\gamma_k \geq 0$ and $f(z) \in T$ of the form (5). The (k, γ)-neighbourhood of a function $f(z)$ defined by*

$$N_{(k,\gamma)}(f) = \left\{ g \in T : g(z) = z - \sum_{k=2}^{\infty} b_k z^k \text{ and } \sum_{k=2}^{\infty} k|a_k - b_k| \leq \gamma \right\} \quad (12)$$

For the identity function $e(z) = z$ we have

$$N_{(k,\gamma)}(e) = \left\{ g \in T : g(z) = z - \sum_{k=2}^{\infty} b_k z^k \text{ and } \sum_{k=2}^{\infty} k|b_k| \leq \gamma \right\}. \quad (13)$$

Theorem 5. *Let*

$$\gamma = \frac{2\beta\xi(1-\alpha)}{(1+\lambda)^n[1+\beta(2\xi-1)]}.$$

Then $TD_\lambda(\alpha,\beta,\xi;n) \subset N_{(k,\gamma)}(e)$.

Proof. Let $f \in TD_\lambda(\alpha,\beta,\xi;n)$ then we have

$$\begin{aligned} 2(1+\lambda)^n[1+\beta(2\xi-1)]\sum_{k=2}^{\infty} a_k & \\ &\le \sum_{k=2}^{\infty}[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]a_k \\ &\le 2\beta\xi(1-\alpha), \end{aligned}$$

therefore

$$\sum_{k=2}^{\infty} a_k \le \frac{\beta\xi(1-\alpha)}{(1+\lambda)^n[1+\beta(2\xi-1)]}, \tag{14}$$

also we have for $|z| < r$

$$|f'(z)| \le 1 + |z|\sum_{k=2}^{\infty} k a_k \le 1 + r\sum_{k=2}^{\infty} k a_k.$$

In view (14), we have

$$|f'(z)| \le 1 + r\frac{2\beta\xi(1-\alpha)}{(1+\lambda)^n[1+\beta(2\xi-1)]}.$$

From above inequalities we get

$$\sum_{k=2}^{\infty} k a_k \le \frac{2\beta\xi(1-\alpha)}{(1+\lambda)^n[1+\beta(2\xi-1)]} = \gamma,$$

therefore $f \in N_{(k,\gamma)}(e)$.

Definition 2. *The function $f(z)$ defined by (5) is said to be a member of the subclass $TD_\lambda(\alpha,\beta,\xi,\zeta;n)$ if there exits a function $g \in TD_\lambda(\alpha,\beta,\xi;n)$ such that*

$$\left|\frac{f(z)}{g(z)} - 1\right| \le 1-\zeta, z \in U, 0 \le \zeta < 1.$$

Theorem 6. *Let* $g \in TD_\lambda(\alpha,\beta,\xi;n)$ *and*

$$\zeta = 1 - \frac{\gamma}{2} d(\alpha,\beta,\xi;n). \tag{15}$$

Then $N_{(k,\gamma)}(g) \subset TD_\lambda(\alpha,\beta,\xi,\zeta;n)$ *where* $n \in N \cup \{0\}$, $\lambda \geq 0$, $0 \leq \alpha < \frac{1}{2}\xi$, $0 < \beta \leq 1$, $1/2 \leq \xi \leq 1$, $0 \leq \zeta < 1$ *and*

$$d(\alpha,\beta,\xi;n) = \frac{(1+\lambda)^n[1+\beta(2\xi-1)]}{(1+\lambda)^n[1+\beta(2\xi-1)] - \beta\xi(1-\alpha)}.$$

Proof. Let $f \in N_{(k,\gamma)}(g)$ then by (14) we have $\sum_{k=2}^{\infty} k|a_k - b_k| \leq \gamma$, then $\sum_{k=2}^{\infty} |a_k - b_k| \leq \frac{\gamma}{2}$. Since $g \in TD_\lambda(\alpha,\beta,\xi;n)$ we have

$$\sum_{k=2}^{\infty} b_k \leq \frac{\beta\xi(1-\alpha)}{(1+\lambda)^n[1+\beta(2\xi-1)]},$$

therefore,

$$\begin{aligned} \left|\frac{f(z)}{g(z)} - 1\right| &< \frac{\sum_{k=2}^{\infty} |a_k - b_k|}{1 - \sum_{k=2}^{\infty} b_k} \\ &\leq \frac{\gamma}{2}\left(\frac{(1+\lambda)^n[1+\beta(2\xi-1)]}{(1+\lambda)^n[1+\beta(2\xi-1)] - \beta\xi(1-\alpha)}\right) \\ &= \frac{\gamma}{2} d(\alpha,\beta,\xi;n) = 1 - \zeta. \end{aligned}$$

Then by Definition 2, we get $f \in TD_\lambda(\alpha,\beta,\xi,\zeta;n)$.

Theorem 7. *Let* $f(z)$ *and* $g(z) \in TD_\lambda(\alpha_1,\beta,\xi;n)$ *be of the form (5) such that* $f(z) = z - \sum_{k=2}^{\infty} a_k z^k$ *and* $g(z) = z - \sum_{k=2}^{\infty} b_k z^k$ *where* $a_k, b_k \geq 0$. *Then the Hadamard product* $h(z)$ *defined by* $h(z) = z - \sum_{k=2}^{\infty} a_k b_k z^k$ *is in the subclass* $TD_\lambda(\alpha_2,\beta,\xi;n)$ *where*

$$\alpha_2 \leq \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)] - 2\beta\xi(1-\alpha_1)^2}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}.$$

Proof. By Theorem 1, we have

$$\sum_{k=2}^{\infty} \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha_1)} a_k \leq 1 \tag{16}$$

and

$$\sum_{k=2}^{\infty} \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha_1)} b_k \leq 1. \tag{17}$$

We have only to find the largest α_2 such that

$$\sum_{k=2}^{\infty} \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha_2)} a_k b_k \leq 1.$$

Now, by Cauchy-Schwarz inequality, we obtain

$$\sum_{k=2}^{\infty} \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha_1)} \sqrt{a_k b_k} \leq 1,$$

we need only to show that

$$\frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha_2)} a_k b_k \leq \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha_1)} \sqrt{a_k b_k}, \quad (18)$$

equivalently,

$$\sqrt{a_k b_k} \leq \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha_1)} \times \frac{2\beta\xi(1-\alpha_2)}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]} \leq \frac{1-\alpha_2}{1-\alpha_1}.$$

But from (18), we have

$$\sqrt{a_k b_k} \leq \frac{2\beta\xi(1-\alpha_1)}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}$$

Consequently, we need to prove that

$$\frac{2\beta\xi(1-\alpha_1)}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]} \leq \frac{1-\alpha_2}{1-\alpha_1}.$$

or equivalently, that

$$\alpha_2 \leq \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)] - 2\beta\xi(1-\alpha_1)^2}{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}.$$

Theorem 8. *Let $f \in TD_\lambda(\alpha,\beta,\xi;n)$ be defined by (5) and c any real number with $c > -1$ than the function $G(z)$ defined as $G(z) = \frac{c+1}{z^c}\int_0^z s^{c-1} f(s)ds$, $c > -1$, also belongs to $TD_\lambda(\alpha,\beta,\xi;n)$.*

Proof. By virtue of $G(z)$ it follows from (5) that

$$G(z) = \frac{c+1}{z^c} \int_0^z \left(s^c - \sum_{k=2}^{\infty} a_k s^{k+c-1} \right) ds$$
$$= z - \sum_{k=2}^{\infty} \left(\frac{c+1}{c+k} \right) a_k z^k.$$

But

$$\sum_{k=2}^{\infty} \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha)} \left(\frac{c+1}{c+k} \right) a_k \leq 1,$$

since $\left(\frac{c+1}{c+k}\right) \leq 1$ and by Theorem 1, so the proof is complete.

Theorem 8. *Let $f \in TD_\lambda(\alpha, \beta, \xi; n)$ be defined by (5) and*

$$F_\mu(z) = (1-\mu)z + \mu \int_0^z \frac{f(s)}{s} ds \quad (\mu \geq 0, z \in U).$$

Then $F_\mu(z)$ is also in $TD_\lambda(\alpha, \beta, \xi; n)$ if $0 \leq \mu \leq 2$.

Proof. Let f defined by (5) then

$$F_\mu(z) = (1-\mu)z + \mu \int_0^z \left(\frac{s - \sum_{k=2}^{\infty} a_k s^k}{s} \right) ds$$
$$= z - \sum_{k=2}^{\infty} \frac{\mu}{k} a_k z^k.$$

By Theorem 1 and since $\left(\frac{\mu}{k} \leq 1\right)$ we have

$$\sum_{k=2}^{\infty} \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha)} \left(\frac{\mu}{k} \right) a_k$$
$$\leq \sum_{k=2}^{\infty} \frac{[1+(k-1)\lambda]^n k[1+\beta(2\xi-1)]}{2\beta\xi(1-\alpha)} \left(\frac{\mu}{2} \right) a_k \leq 1,$$

then $F_\mu(z)$ is in $TD_\lambda(\alpha, \beta, \xi; n)$.

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S. B. Joshi: Department of Mathematics, Walchand College of Engineering, Sangli (M.S), India 416 415.
E-mail: joshisb@hotmail.com.

N. D. Sangle: Department of Mathematics, Annasaheb Dange College of Engineering, Ashta, Sangli (M.S), India 416 301.
E-mail: navneet_sangle@rediffmail.com.